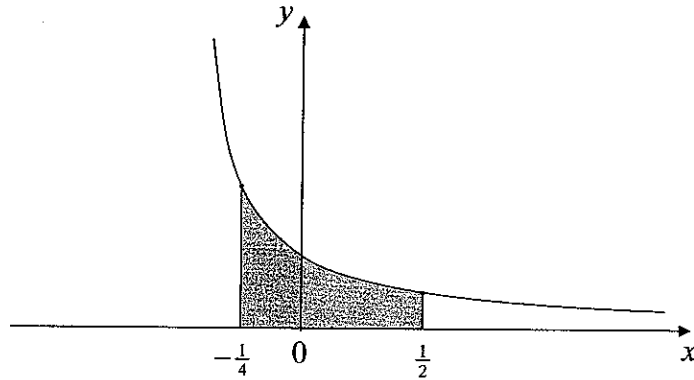


2.

Figure 1



The curve with equation $y = \frac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the x -axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x -axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

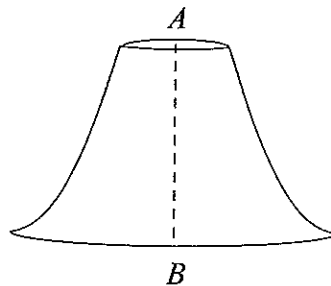


Figure 2 shows a paperweight with axis of symmetry AB where $AB = 3$ cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)



3. A curve has parametric equations

$$x = 7 \cos t - \cos 7t, \quad y = 7 \sin t - \sin 7t, \quad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t . You need not simplify your answer. (3)

(b) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$.
Give your answer in its simplest exact form. (6)



4. (a) Express $\frac{2x-1}{(x-1)(2x-3)}$ in partial fractions. (3)

(b) Given that $x \geq 2$, find the general solution of the differential equation

$(2x-3)(x-1) \frac{dy}{dx} = (2x-1)y.$ (5)

(c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = f(x)$. (4)

Horizontal lines for writing the solution to the differential equation.



6. (a) Given that $y = 2^x$, and using the result $2^x = e^{x \ln 2}$, or otherwise, show that $\frac{dy}{dx} = 2^x \ln 2$. (2)

(b) Find the gradient of the curve with equation $y = 2^{(x^2)}$ at the point with coordinates (2,16). (4)

Lined area for student response.



7. The point A has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point B has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin O .

(a) Find the position vector of the point C , with position vector \mathbf{c} , given by

$$\mathbf{c} = \mathbf{a} + \mathbf{b}. \tag{1}$$

(b) Show that $OACB$ is a rectangle, and find its exact area. (6)

The diagonals of the rectangle, AB and OC , meet at the point D .

(c) Write down the position vector of the point D . (1)

(d) Find the size of the angle ADC . (6)



Question 7 continued

Lined writing area for the answer.



8.

$$I = \int_0^5 e^{\sqrt{3x+1}} dx.$$

- (a) Given that $y = e^{\sqrt{3x+1}}$, complete the table with the values of y corresponding to $x = 2, 3$ and 4 .

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I , giving your answer to 4 significant figures.

(3)

- (c) Use the substitution $t = \sqrt{3x+1}$ to show that I may be expressed as $\int_a^b kte' dt$, giving the values of a, b and k .

(5)

- (d) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)



